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TECHNICAL NOTES

ANALYSIS OF 1–2 SPLIT FLOW HEAT EXCHANGER

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NOMENCLATURE

A	surface area of heat exchanger [m^2]
c	specific heat of tube side fluid [kcal kg^{-1} $°C^{-1}$]
C	specific heat of shell side fluid [kcal kg^{-1} $°C^{-1}$]
K_1, K_2, K_3	constants, equation (17)
K'_1, K'_2, K'_3	constants, equation (9)
L	length of the exchanger [m]
Q	heat transfer rate [kcal h^{-1}]
R	parameter, equation (5)
t	temperature of tube side fluid [°C]
T	temperature of shell side fluid [°C]
Δt	logarithmic mean temperature difference [°C]
U	overall heat transfer coefficient [kcal m^{-2} h^{-1} $°C^{-1}$]
w	flow rate of tube side fluid [kg h^{-1}]
W	flow rate of shell side fluid [kg h^{-1}]
x	linear coordinate

Subscripts

1	inlet
2	outlet
A	section A
B	section B
PCF	parallel counterflow
SF	split flow
x	value at or upto location x

Superscripts

$\bar{}$	value at split junction
$'$	outlet of section A
$''$	outlet of section B

1. INTRODUCTION

DUE TO their improved heat transfer and pressure drop characteristics, split flow shell and tube heat exchangers are often used in power and process industries, especially when the allowable shell side pressure drop is small.

The heat transfer in a split flow heat exchanger with one tube pass and one shell pass has been analysed [1]. The shell side and tube side fluid temperatures were obtained as a function of the area of the heat exchanger. In the present paper, the heat transfer process in a split flow exchanger with one shell pass and two tube passes is studied. The performance of such an exchanger is compared with that of a 1–2 parallel-counterflow heat exchanger. Figure 1 shows the typical tube arrangement of a 1–2 split flow heat exchanger. The temperature relationships are indicated in Fig. 2. Though it was indicated [2] that the method of solution is by trial and error, no attempt has so far been made towards the detailed analysis which forms the objective of this paper.

2. ANALYSIS

Referring to Fig. 2, section A, the heat balance at a point x on the exchanger length gives

$$wc\,dt_{1x} = \frac{U\,dA}{2}(T_x - t_{1x}), \tag{1}$$

$$wc\,dt_{2x} = -\frac{U\,dA}{2}(T_x - t_{2x}). \tag{2}$$

Heat balance from $L = 0$ to $L = x$ yields

$$\frac{WC}{2}(T_x - T'_2) = wc[(t_2 - t_1) - (t_{2x} - t_{1x})]. \tag{3}$$

Differentiating equation (3) gives

$$\frac{WC}{2}\,dT_x = wc(dt_{1x} - dt_{2x}). \tag{4}$$

Combining equations (1), (2) and (4) gives

$$\frac{dT_x}{dA} - \frac{2URT_x}{wc} + \frac{UR}{wc}(t_{1x} + t_{2x}) = 0 \tag{5}$$

where $R = wc/WC$.

Differentiating w.r.t. A gives

$$\frac{d^2T_x}{dA^2} - \frac{2UR}{wc}\frac{dT_x}{dA} + \frac{UR}{wc}\left[\frac{dt_{1x}}{dA} + \frac{dt_{2x}}{dA}\right] = 0. \tag{6}$$

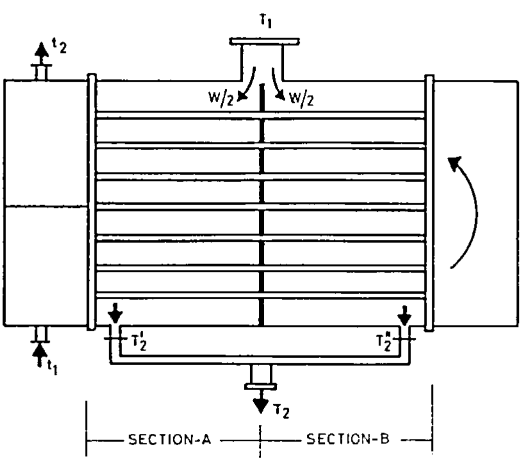

FIG. 1. 1–2 split flow heat exchanger.

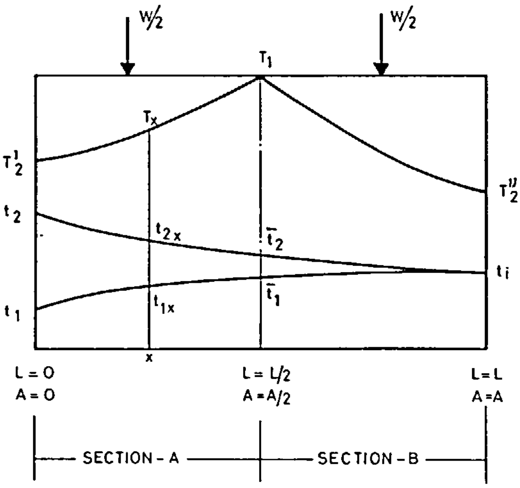

FIG. 2. Temperature relations in 1–2 split flow.

Substituting for $(\mathrm{d}t_{1x}/\mathrm{d}A)$ and $(\mathrm{d}t_{2x}/\mathrm{d}A)$ from equations (1) and (2) into equation (6) yields

$$\frac{\mathrm{d}^2T_x}{\mathrm{d}A^2}-\frac{2UR}{wc}\frac{\mathrm{d}T_x}{\mathrm{d}A}+\frac{2U^2R}{(2wc)^2}(t_{2x}-t_{1x})=0. \quad (7)$$

Substituting for $(t_{2x}-t_{1x})$ in equation (7) from equation (3) and differentiating equation (7) w.r.t. A gives

$$\frac{\mathrm{d}^3T_x}{\mathrm{d}A^3}-\frac{2UR}{wc}\frac{\mathrm{d}^2T_x}{\mathrm{d}A^2}-\left[\frac{U}{2wc}\right]^2\frac{\mathrm{d}T_x}{\mathrm{d}A}=0. \quad (8)$$

Solution of equation (8) is given by

$$T_x = K_1'+K_2'\,\mathrm{e}^{\frac{UAx}{wc}[R+(R^2+\frac{1}{4})^{1/2}]}+K_3'\,\mathrm{e}^{\frac{UAx}{wc}[R-(R^2+\frac{1}{4})^{1/2}]}. \quad (9)$$

Substituting for $(\mathrm{d}T_x/\mathrm{d}A)$ and $(\mathrm{d}^2T_x/\mathrm{d}A^2)$ as obtained from equation (9) and for $(t_{2x}-t_{1x})$ from equation (3) into equation (7) yields

$$K_1' = 2T_1-T_2'' = 2(T_1-T_2)+T_2'. \quad (10)$$

From equation (9) at $A = 0$,

$$T_x = T_2' = K_1'+K_2'+K_3'$$

Thus

$$K_2'+K_3' = T_2'-K_1 = -2(T_1-T_2). \quad (11)$$

Since at $A = 0$,

$$T_x = T_2', \quad t_{1x} = t_1,$$

and

$$t_{2x} = t_2$$

and equating the value of $(\mathrm{d}T/\mathrm{d}A)$ at $A = 0$ from equations (5) and (9),

$$[R+(R^2+\tfrac{1}{4})^{1/2}]K_2'+[R-(R^2+\tfrac{1}{4})^{1/2}]K_3' = 2RT_2'-R(t_1+t_2). \quad (12)$$

Solving for K_2' and K_3' from equations (11) and (12)

$$K_2' = \frac{[R-(R^2+\frac{1}{4})^{1/2}](2T_1-T_2'-T_2'')+2RT_2'-R(t_1+t_2)}{2(R^2+\frac{1}{4})^{1/2}} \quad (13)$$

and

$$K_3' = \frac{-[R+(R^2+\frac{1}{4})^{1/2}](2T_1-T_2'-T_2'')-2RT_2'+R(t_1+t_2)}{2(R^2+\frac{1}{4})^{1/2}} \quad (14)$$

The above analysis when repeated for section B gives

$$\frac{\mathrm{d}^2T_x}{\mathrm{d}A^2}+\frac{2UR}{wc}\frac{\mathrm{d}T_x}{\mathrm{d}A}-\frac{2U^2R}{(2wc)^2}(t_{2x}-t_{1x})=0, \quad (15)$$

$$\frac{\mathrm{d}^3T_x}{\mathrm{d}A^3}+\frac{2UR}{wc}\frac{\mathrm{d}^2T_x}{\mathrm{d}A^2}-\left(\frac{U}{2wc}\right)^2\frac{\mathrm{d}T_x}{\mathrm{d}A}=0. \quad (16)$$

The solution of equation (16) is given by

$$T_x = K_1+K_2\,\mathrm{e}^{-\frac{UAx}{wc}[R+(R^2+\frac{1}{4})^{1/2}]}+K_3\,\mathrm{e}^{-\frac{UAx}{wc}[R-(R^2+\frac{1}{4})^{1/2}]}. \quad (17)$$

Substituting for T_x, $(\mathrm{d}T_x/\mathrm{d}A)$ and $(\mathrm{d}^2T_x/\mathrm{d}A^2)$ as obtained from equation (17) into equation (15) yields

$$K_1 = T_2''. \quad (18)$$

At $A = A/2$ for section B (i.e. at $A = A$ for the total exchanger) substituting for K_1 from equation (18) into equation (17) gives

$$-K_2\,\mathrm{e}^{-\frac{UA}{2wc}[R+(R^2+\frac{1}{4})^{1/2}]} = K_3\,\mathrm{e}^{-\frac{UA}{2wc}[R-(R^2+\frac{1}{4})^{1/2}]}$$

i.e.

$$\frac{-K_2}{K_3} = \mathrm{e}^{\frac{UA}{2wc}[2(R^2+\frac{1}{4})^{1/2}]},$$

or

$$\frac{UA}{wc} = \frac{1}{(R^2+\frac{1}{4})^{1/2}}\ln\left[-\frac{K_2}{K_3}\right]. \quad (19)$$

At $A = 0$ for section B, equation (17) gives

$$T_1 = T_2''+K_2+K_3,$$

or

$$K_2+K_3 = T_1-T_2''. \quad (20)$$

Equating the value of $(\mathrm{d}T_x/\mathrm{d}A)$ at $A = 0$ for section B in a manner similar to that of section A gives

$$-K_2[R+(R^2+\tfrac{1}{4})^{1/2}]-K_3[R-(R^2+\tfrac{1}{4})^{1/2}] = -2RT_1+R(\bar{t}_1+\bar{t}_2). \quad (21)$$

Solving for K_2 and K_3 from equations (20) and (21)

$$K_2 = \frac{-[R-(R^2+\frac{1}{4})^{1/2}](T_1-T_2'')+2RT_1-R(\bar{t}_1+\bar{t}_2)}{2(R^2+\frac{1}{4})^{1/2}} \quad (22)$$

and

$$K_3 = \frac{[R+(R^2+\frac{1}{4})^{1/2}](T_1-T_2'')-2RT_1+R(\bar{t}_1+\bar{t}_2)}{2(R^2+\frac{1}{4})^{1/2}}. \quad (23)$$

Thus

$$\left[\frac{-K_2}{K_3}\right] = \frac{[R-(R^2+\frac{1}{4})^{1/2}](T_1-T_2'')-2RT_1+R(\bar{t}_1+\bar{t}_2)}{[R+(R^2+\frac{1}{4})^{1/2}](T_1-T_2'')-2RT_1+R(\bar{t}_1+\bar{t}_2)}. \quad (24)$$

The ratio of heat transfer rates in the two halves of the split flow exchanger can be written as

$$n = \frac{Q_A}{Q_B} = \frac{T_1-T_2'}{T_1-T_2''} = \left[\frac{t_2-t_1}{\bar{t}_2-\bar{t}_1}\right]-1. \quad (25)$$

3. SOLUTION PROCEDURE

From Fig. 1 it is seen that

$$T_2 = \frac{T_2'+T_2''}{2}. \quad (26)$$

Heat balance for section B gives

$$T_1 - T_2'' = 2R(\bar{t}_2 - \bar{t}_1). \tag{27}$$

Equating the expressions for (dT/dA) at the split junction, as obtained from equations (5) and (9), one obtains

$$2RT_1 - R(\bar{t}_1 + \bar{t}_2) = K_2'[R + (R^2 + \tfrac{1}{4})^{1/2}]\, e^{\frac{UA}{2wc}[R + (R^2 + \frac{1}{4})^{1/2}]} + K_3'[R - (R^2 + \tfrac{1}{4})^{1/2}]\, e^{\frac{UA}{2wc}[R - (R^2 + \frac{1}{4})^{1/2}]}. \tag{28}$$

First, a value of n is assumed. As shown later, the range of n is limited by the practicability of the solution for given inlet and outlet temperatures. For the assumed value of n, $(\bar{t}_2 - \bar{t}_1)$ and T_2'' are found from equations (25) and (27). Next, a value is assumed for the sum $(\bar{t}_1 + \bar{t}_2)$ and $(-K_2/K_3)$ is evaluated from equation (24). Thus (UA/wc) is calculated using equation (19). Using this value of (UA/wc), it is verified whether equation (28) is satisfied. If not, with a new value for the sum $(\bar{t}_1 + \bar{t}_2)$, the procedure is repeated until equation (28) is satisfied. This gives the solution, i.e. the values of T_2', T_2'', $\bar{t}_1$ and $\bar{t}_2$ for given T_1, T_2, t_1, t_2 and n. This trial and error procedure for a given n agrees, in principle, with that indicated in ref. [2].

4. RESULTS AND DISCUSSION

Results for a 1–2 split flow heat exchanger with $T_1 = 300°C$, $T_2 = 200°C$, $t_1 = 100°C$ and $t_2 = 150°C$ are presented in Table 1 for n = 0.90, 0.95, 1.00, 1.05 and 1.10.

Table 1

n	T_2'	T_2''	$\bar{t}_1$	$\bar{t}_2$
0.90	205.264	194.736	102.592	128.908
0.95	202.56	197.44	108.28	133.92
1.00	200	200	113.75	138.75
1.05	197.56	202.44	118.905	143.295
1.10	195.24	204.76	123.845	147.655

A close examination of Table 1 reveals that for n slightly less than 0.9, $\bar{t}_1$ will get reduced to a value lower than t_1 (100°C) which is not practicable for the given data. Similarly for n greater than 1.1, $\bar{t}_2$ will become greater than t_2 (150°C) which is again impracticable. Thus, in the given example, n can be varied only between a value slightly less than 0.9 and a value slightly greater than 1.1. The logarithmic mean temperature difference in each case is calculated as follows:

$$\Delta t_{SF} = \frac{t_2 - t_1}{(UA/wc)_{SF}}. \tag{29}$$

The values of Δt_{SF} obtained for n = 0.9, 0.95, 1.0, 1.05 and 1.1 are compared with Δt obtainable in a 1–2 parallel counterflow as given by equation (7.37) of ref. [2]. The comparison presented in Fig. 3 shows that for equal amounts of heat transferred in the two sections A and B; i.e. for $n = 1$, the split flow arrangement has a $\Delta t'$ 0.05% more, or it requires an exchanger surface area 0.05% less than that of a 1–2 parallel counterflow arrangement, operating between the same temperature limits T_1, T_2, t_1, t_2 and transferring the same heat duty. The split flow arrangement has a greater advantage in the range $n \leqslant 1.0$, wherein it requires a surface area less than the 1–2 PCF arrangement, e.g. about 1.5% less at $n = 0.90$. With increasing value of n, the advantage is lost, e.g. it requires 1.15% more surface area at $n = 1.10$. It is thus clear that the best performance of split flow arrangement is obtained in the range $0.9 \leqslant n \leqslant 1.0$ for the given example. Its performance is inferior to that of 1–2 PCF exchanger in the range $1.0 < n \leqslant 1.10$. In view of the complexity of construction of the split flow arrangement as compared to 1–2 PCF arrangement, it is not considered worthwhile to choose a split flow arrangement in the present example, wherein the maximum possible saving in surface area is only 1.5% (at $n = 0.9$). However, in cases where the allowable shell fluid pressure drop is severely limited, it is preferable to use a split flow arrangement wherein the shell fluid pressure drop would be approximately one-eighth of that in a conventional exchanger [2], though it entails providing slightly more surface area in some cases, e.g. for $n > 1.0$ in the above example.

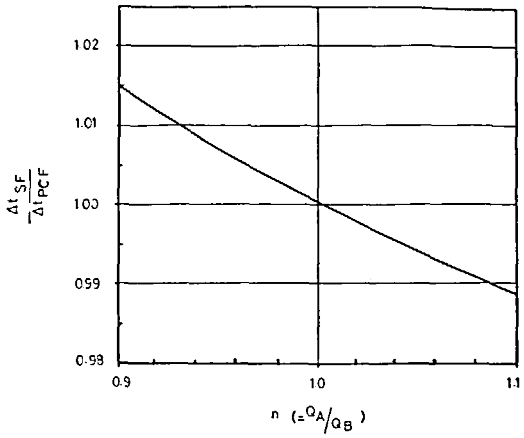

FIG. 3. Comparison with 1–2 parallel counterflow.

5. CONCLUSIONS

(1) The heat transfer in a 1–2 split flow heat exchanger is analysed. Expressions are given for shell fluid temperature as a function of exchanger length or area.

(2) A trial and error procedure is presented to solve for unknown temperatures for a given n.

(3) The split flow arrangement has a better thermal performance in the range $0.9 \leqslant n \leqslant 1.0$, while the 1–2 PCF exchanger performs better in the range $1.0 < n \leqslant 1.1$.

(4) The use of split flow arrangement is recommended in cases where the allowable shell fluid pressure drop is severely limited.

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